

Reflection on (and in) Strunk's *Tonnetz*¹

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INTRODUCTION

In 2011, during the national meeting of the Society for Music Theory in Minneapolis, I met privately with Steven Strunk and showed him my analysis of Wayne Shorter's "Fee-Fi-Fo-Fum" that uses a *Tonnetz* to map the contour of the melody (Figure 1). When describing the motion of the triangle (shown in Figure 1 as arrows), I repeatedly used the word "flip." One of Strunk's criticisms of my analysis was the use of the term "flip." He said that nowhere in the world of geometric operations do we find the word "flip."² Instead, he advocated the use of terms such as "translation," "rotation," and "reflection," borrowed from the field of transformational geometry, to describe operations on the neo-Riemannian *Tonnetz*.³ Strunk's insistence on using geometric terms suggests that he comprehended the neo-Riemannian *Tonnetz* to be geometric in nature rather than group-theoretic, as they were originally conceived.

¹ I presented an earlier version of this paper at the 2013 meeting of the Music Theory Society of the Mid-Atlantic. I would like to thank the following people for the invaluable input in shaping this article: Keith Salley, Keith Waters, and the anonymous reviewers.

² Strunk's 2003 article "Wayne Shorter's *Yes and No: An Analysis*" cites two examples (Cohn 1997 and Morris 1998) to show his frustrations over theorists' use of terms, such as "translation" and "flip," without proper definitions (2003, 49n21). The term "flip," in particular, has been defined differently depending on the theorist. Perhaps the most famous example in the neo-Riemannian literature that uses the word "flip" is David Lewin's article "Cohn Functions" (1996). Lewin's formal definition of the term *Cohn flip* is outside the scope of this article. Instead, I cite one of his examples of the Cohn flip to demonstrate how the term is used. "...the pcset {C,E,G} can be Cohn-flipped into its inverted form {C,E♭,G}, casting out E and substituting the adjacent pitch class E♭" (181). Edward Gollin also uses the term "edge-flip" and "vertex-flip" to describe his neo-Riemannian operations on three-dimensional *Tonnetze* (1998). More recently, Dmitri Tymoczko use the word "flip" throughout his article while acknowledging that the "flips" in neo-Riemannian literature are "reflections" in geometry (2012, 1).

³ Figure 2 shows examples of geometric translation, reflection, and rotation.

a single P, R, or L operation (Figure 3 and 4).⁴ The three neo-Riemannian operations P, R, and L, were originally defined as operations on triads only. Therefore, Strunk's application of these operations on chords other than major and minor triads is in itself a significant extension of the conventional neo-Riemannian operations. He achieved this by taking a geometric shape created by connecting on the Tonnetz the notes that correspond to a chord (for example, a parallelogram for seventh chords) and reflecting them over horizontal, vertical, and diagonal axes. Strunk redefines the neo-Riemannian operations as geometric reflections, which, in turn, made the Tonnetz much more of a geometric grid than a representation of group-theoretic operations. This I consider a "next step" in the ontological development of musical space.⁵ Strunk's reference to a college geometry text when introducing the Tonnetz also serves as an evidence of the geometric orientation of his method (Strunk 2003, 49–50).⁶

One discrepancy between the conventional method and Strunk's stands out. It is between the change of set class with Strunk's P, R, and L operations on major seventh and minor seventh chords (Figure 4) and the preservation of set class in David Lewin's original definition of P, R, and L as "contextual inversions" (1982; 1987, 178). A detailed discussion of contextual inversion will follow; for now, it is important to note that contextual inversion is a type of inversion (i.e., a subset of TnI operations), therefore the set class is unaffected after the application. When Strunk's method is applied to a major or minor triad (Figure 3), the result is identical to the conventional method because the set class for both triads is the same: 3–11 (037). In contrast, the set classes are different between major and minor seventh chords, 4–20 (0158) and 4–26 (0358) respectively; yet, Strunk related the two chord types with a single operation just like their triadic counterpart (Figure 4). Since P, R, and L are originally defined

⁴ Ninth chords are omitted from Figures 3 and 4. Here, Strunk's labeling "Rf(D)" denotes reflection over diagonal axis, "Tr" denotes translation, and "Ro(180)" denotes rotation of 180 degrees. Although he does entertain the possibility of reflecting over the vertical and horizontal axes, written as "Rf(V)" and "Rf(H)," he only uses reflection over the diagonal axes in the article in his analysis of Wayne Shorter's "Yes and No." (49). He does not differentiate the reflection over the diagonal axis and the anti-diagonal axis. For another approach to modeling ninth chords on a Tonnetz, see Waters and Williams 2010.

⁵ The change in foundation from group theory to geometry has some affinities with the later development of neo-Riemannian theory. Dmitri Tymoczko clarifies the tension between the two in his comment on Julian Hook's article (Hook 2007; Tymoczko 2008). He claims that the success of Hook's work on cross-type transformations suggests that "groups of transformations may not suffice for modeling musical intervals" (2008, 164).

⁶ Strunk cites Lewin (1987), Cohn (1997), Robert Morris (1998), Hyer (1989), and Henry Klumpenhouwer (2000) for referencing neo-Riemannian theory and cites a geometry textbook by Howard Eves (1963, 123–4) for referencing the geometric transformations.

as the transformations on major and minor triads, Strunk explains that extending the three transformations to consider similarly functioning major seventh and minor seventh chords is “reasonable for analysis of jazz because, in jazz, the degree to which a chord is extended is variable and is determined on an *ad hoc* basis by the rhythm section” (48). Despite the fact that the set classes for major and minor seventh chords are different, Strunk advocated that the way P, R, and L connects major and minor triad should be extended to major and minor seventh (and also ninth) chords because of jazz’s performance practice.

This paper reflects on Strunk’s use of the Tonnetz, and more specifically, how P, R, and L operations became geometric reflections in the Tonnetz in Strunk’s analysis of post-bop jazz.⁷ The Tonnetz, in its conventional use, “provides a canonical geometry of *modeling*” the three contextual inversions and D (for dominant) transformation (Cohn 1998, 172; italics added; Figure 5). The fact that the four transformations are “modeled” on the Tonnetz presupposes that these transformations are applied to major and minor triads prior to the graphic representation. In other words, the conventional method is a two-step process of (1) applying the transformations and (2) representing the application. This procedure prevents the transformations from being geometrically interpreted because the application precedes their graphic representation. As we shall see, Strunk’s method does not follow this procedure because Strunk defines the transformations as geometric in nature. Strunk’s re-envisioning of P, R, and L transformations as geometric reflections not only provides a way to relate major seventh and minor seventh chords on the Tonnetz but also show a notable conceptual difference between the conventional method and Strunk’s as to what a chord is. Close examination of the methodological differences will shed light on how jazz’s performance practice differentiates jazz theory from theories of more conventional tonal music and will allow us to explore new possibilities of treating the Tonnetz as Strunk did. In this article, I will investigate the methodological difference and extend Strunk’s method further by entertaining the possibility of representing Z-related pairs on the Tonnetz.

⁷ Strunk’s posthumous article (2016) applies similar geometric methods to Chick Corea’s compositions of the 1960s.

Figure 3. P, R, and L for major and minor triads; a = major, b = minor (based on Strunk 2003, Example 6).

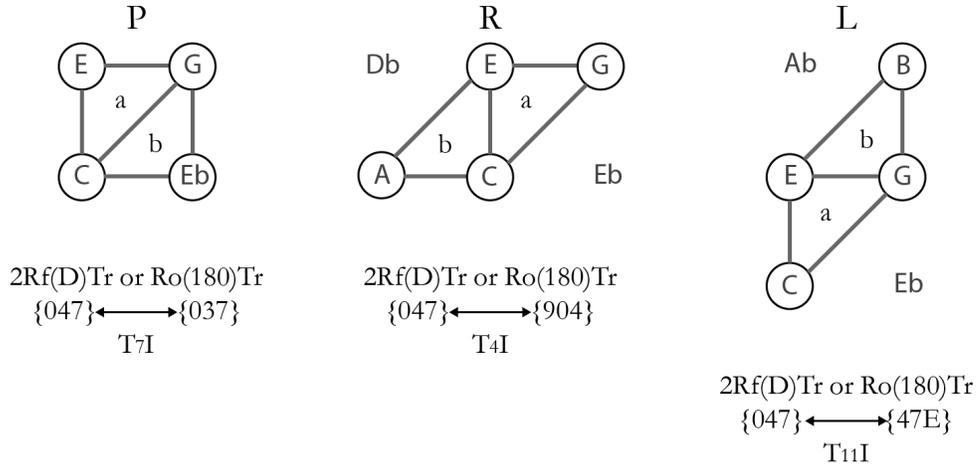


Figure 4. P, R, and L for major seventh and minor seventh chords; a = major, b = minor (based on Strunk 2003, Example 7).

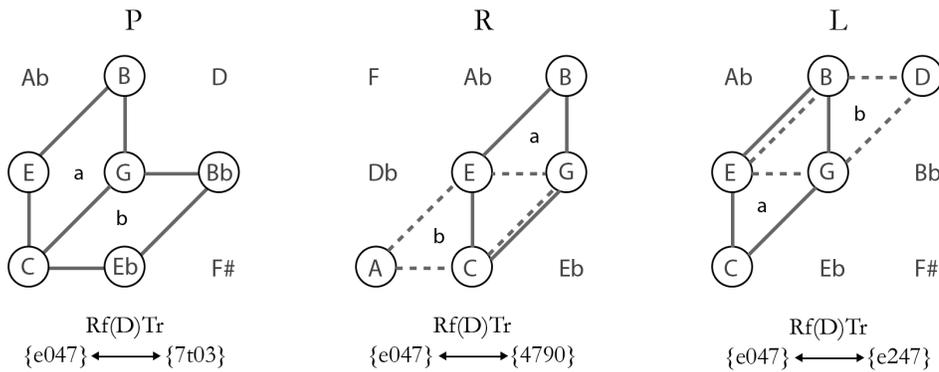
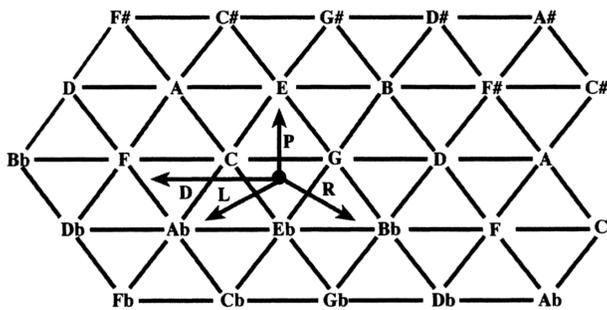


Figure 5. The P, R, L, and D transformations on the Tonnetz (Cohn 1998, 172, Figure 2).



METHODOLOGICAL DIFFERENCE

The above suggests that the difference between the conventional method and Strunk's hinges on whether the three transformations are treated as contextual inversions or as geometric reflections on the Tonnetz. Alternately, the difference between the two methods can be determined in light of another concept known as "parsimonious voice-leading," which focuses on the retention of two common tones and stepwise motion of the remaining note as a result of contextual inversion (Cohn 1996, 1997). By way of clarifying what is conventional in the conventional use of the Tonnetz, this section begins by reviewing how David Lewin originally conceives the three transformations as contextual inversions and how Richard Cohn reframes them regarding their parsimonious voice leading. Afterward, the conventional method and Strunk's will be compared by applying both contextual inversion and Strunk's transformation on a major seventh chord to highlight different outcomes. This section ends with a discussion of how Strunk's method reflects jazz's performance practice.

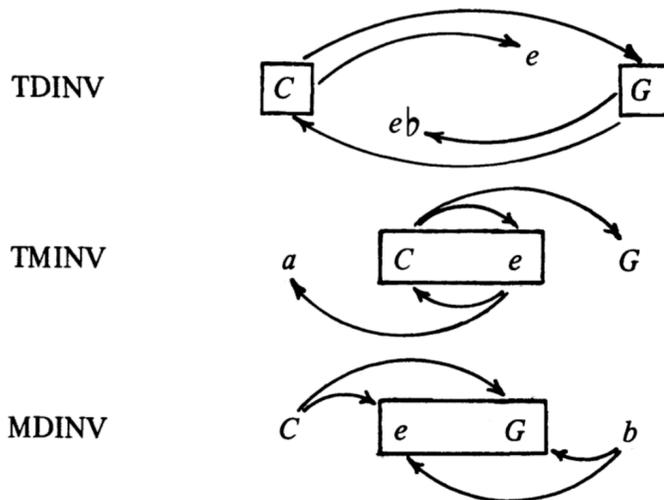
As summarized in "Introduction to Neo-Riemannian Theory," contextual inversion "inverts a triad, mapping major and minor triads to each other" while "the inversional axis is defined in relation to the triad's component pitch classes rather than as a fixed point in pitch-class space" (Cohn 1998, 170). The figure from Lewin's 1982 article illustrates three contextual inversions, reproduced here in Figure 6 (52).⁸ Lewin originally named the three transformations as TDINV (tonic-dominant inversion), TMINV (tonic-mediante inversion), and MDINV (mediante-dominant inversion) to denote the two component pitch classes that are invariant in each transformation. (Each of the three invariant dyads is indicated with a box in Figure 6.) Lewin's example uses a C major triad to show how the three transformations generate three different minor triads (C minor, A minor, and E minor respectively). In each example, the arrows above the pitch classes show how a C major triad is generated with a combination of "tonic pitch class" C (indicated as the origin of the arrows), "dominant interval" pointing to the triadic fifth, and "mediante interval" pointing to the triadic third (Lewin 1982, 23).⁹ The arrows below the pitch classes show how minor triads are generated

⁸ Figure 6 is simplified from Lewin's original version. In the original, the figure includes dotted lines that connect the two component pitch classes that form perfect fourth (G to C, E to A, and B to E). The original figure also has what Lewin calls "the conventional Riemann symbols" (a concept that will not concern us here).

⁹ Lewin uses "T" for tonic pitch class, "d" for dominant interval, and "m" for mediant interval in what he calls the "Riemann System," and it is notated as (T,d,m) (26). P-transformation of the Riemann System (T,d,m) (TDINV from Lewin's original writing) is notated as (T+d, -d, -m)

with the same combination of the arrows (dominant and mediant intervals) but in the opposite direction from the triadic fifth, which now functions as the origin of the arrows (the fifths for each example are G, E, and B respectively in Figure 6). For example, the C minor triad generated by TDINV on a C major triad is described as tonic pitch class G, a dominant interval (i.e., perfect fifth) in the opposite direction to C, and a mediant interval (i.e., major third) in the opposite direction to E \flat . The three transformations TDINV, TMINV, and MDINV are later renamed as PAR, REL, and LT (respectively) by Lewin (1987, 175–80); and PAR, REL, and LT are abbreviated to P, R, and L and plotted on the Tonnetz by Brian Hyer (1989).¹⁰ Figure 7 plots the three contextual inversions—now written as P, R, and L—on staves, accompanied by Lewin’s figures, which are now oriented vertically to match the orientation of the staff notation. The empty note heads correspond to Lewin’s boxed pitch classes (i.e., the three invariant dyads) while the filled-in note heads are used to denote the remaining member of each triad.

Figure 6. Three contextual inversions (Lewin 1982, 52, Figure 12)



where “T+d” denotes the triadic fifth, “-d” and “-m” denote the perfect fifth and major third intervals downward from the T+d (33).

¹⁰ Richard Cohn’s “Introduction to Neo-Riemannian Theory” (1998), Henry Klumpenhouwer’s “Remarks on American Neo-Riemannian Theory” (2000), and David Kopp’s chapter “Riemann’s Legacy and Transformation Theories” in his book *Chromatic Transformations in Nineteenth-Century Music* (2002, 135–155) provide detailed accounts of the early development of neo-Riemannian theory and its historical precursors.

Figure 7. Contextual inversion of a C-major triad represented as staff notation.

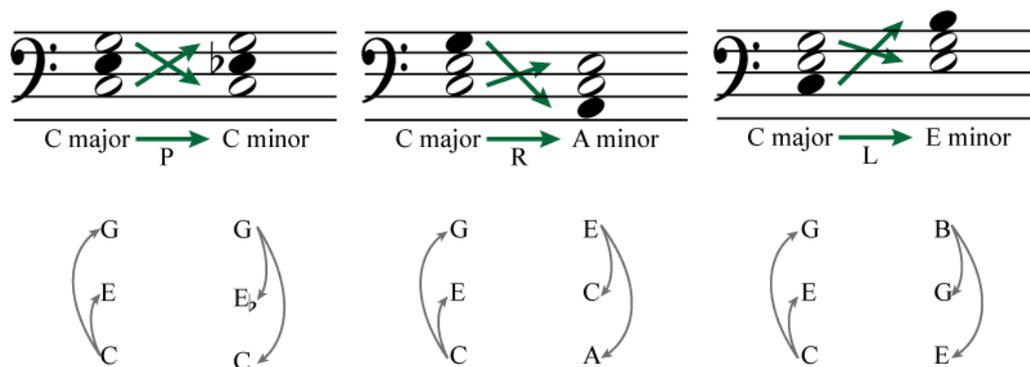
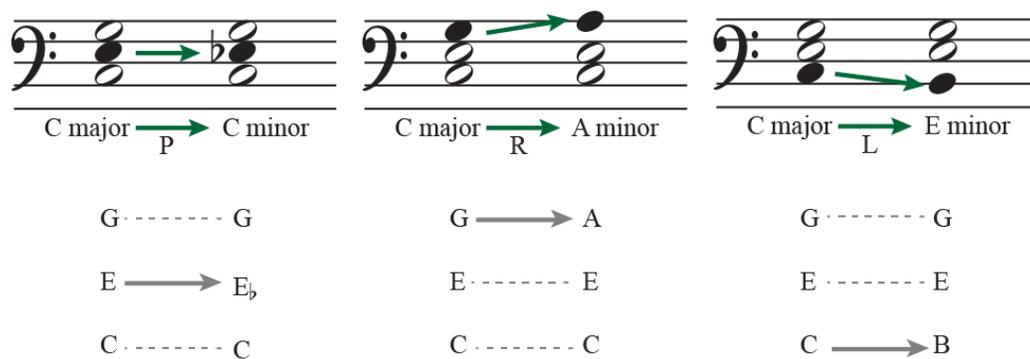


Figure 8. Parsimonious Voice-Leading of a C-major triad.



Tracing the origin of P, R, and L transformations helps us understand an important distinction between treating the three transformations as inversions and treating them as a result of parsimonious voice-leading. As contextual inversions, P, R, and L transformations treat a chord as an object that cannot be taken apart. Lewin describes this class of transformations as “something one does to a *Klang* to obtain another *Klang*” (1987, 177). However, because of the dyadic invariance for each transformation, the net change after each transformation is a stepwise motion of the remaining note.¹¹ This stepwise motion gave rise to the concept of parsimonious voice leading (Cohn 1996; 1997). Instead of considering the movement of all three notes as implied by an inversion of a triad, parsimonious voice leading finds an economical way to relate major and minor triads by the stepwise motion of just one note. Parsimonious voice leading, therefore, is

¹¹ Richard Cohn calls this dyadic invariance “double common-tone retention” (1997, 1).

a concept that treats the two invariant pitch classes not as switching places but as staying where they are.¹² As transformations that feature parsimonious voice leading, P, R, and L can each be recontextualized as a motion of a single voice where P moves the triad's third, R moves the fifth, and L moves the root when changing the quality of a triad from major to minor (Figure 8). Similarly—and with traditional dualist notions notwithstanding—we may consider R as a simple motion of a triad's root, and L as the motion of a triad's fifth when progressing from minor to major. In other words, by conceptualizing these transformations in terms of parsimonious voice leading, we can define these transformations without referring to their inversional origin (Cohn 1997, 11–12).¹³

The conventional Tonnetz representation of the three transformations does not distinguish whether the represented transformations are a result of contextual inversions or parsimonious voice leadings. The representation neither perfectly conforms to nor blatantly contradicts both contextual inversions and parsimonious voice leading because it simplifies the three transformations (regardless of their conceptual origin) with a single arrow. As shown earlier in Figure 2, each of the three transformations (excluding the D transformation) is represented as an inversion of a triangle, which resembles the contextual inversion of a triad. What is not apparent in the Tonnetz representation is the reciprocal mapping of the two pitch classes of the invariant dyads. For example, in Figure 2, the P transformation on the Tonnetz shows the mapping of E \flat to E but does not show the G to C and C to G mappings. Similarly, each transformation on the Tonnetz (again, excluding the D transformation) features retention of the common edge, which corresponds to the double common-tone retention of parsimonious voice-leading. On the other hand, parsimonious voice leading, a concept that is independent of inversion, is nevertheless represented as inversions of a triangle on the Tonnetz. In summary, the Tonnetz only shows the common features of contextual inversions, parsimonious voice-leading, and triads involved in the transformations. Used conventionally, it only illustrates the three transformations and betrays nothing of their conceptual grounding.

¹² Cohn 1997 presents an example of “the PRL family” on a C minor triad that treats the common tones as stationary rather than switching places (1–2).

¹³ Contextual inversion and parsimonious voice leading similarly provide distinct rationales when neo-Riemannian theory is extended to include seventh chords. For example, Edward Gollin’s “3-D Tonnetz” relates two seventh-chord types (dominant and half-diminished) via contextual inversion (1998) while Adrian Childs relates the same two seventh-chord types by double common-tone retention and stepwise motion of the remaining two notes (1998). One of the reasons why dominant seventh and half-diminished seventh chords are used in each of the extensions is that the two types share the same set class 4-27 (0268).

Strunk departs from this convention of treating Tonnetz representations as an illustration by introducing the three transformations as done on the Tonnetz. He writes, "When graphed on the Tonnetz, the operations [the three transformations] *take on geometric characteristics*" (49; italics added). Strunk defines the three transformations as follows: "Each operation changes the type of triad, either from major to minor or vice versa. The parallel operation holds interval class 5 invariant between the two chords, the relative holds interval class 4, and the leading-tone exchange...holds interval class 3" (47).¹⁴ Instead of referring to group-theoretic origins, he defined the neo-Riemannian operations mainly according to interval-class invariance. Although both contextual inversion and parsimonious voice-leading feature interval-class invariance, the invariance is best understood regarding its Tonnetz representation which accompanies his definition (Figure 3). In the Tonnetz, the three interval classes correspond to the three line segments connecting each pitch class of a triangle. The figure clearly shows which of the line segments are held invariant in each of the transformations. With this visualization, the phrase "holding interval classes invariant" acquires the more geometric connotation of "(performing geometric reflection) holding the shared edge of the triangles invariant." Without referring to contextual inversion and parsimonious voice leading, Strunk's definition encourages readers to conceptualize the three transformations more geometrically.

When applied to major and minor triads, the three geometric reflections do not seem to result in an outcome that is different from one achieved by conventional methods. The discrepancy between the conventional method and Strunk's becomes evident, however, when contextual inversion is applied on seventh chords.¹⁵ Figure 9 shows all possible contextual inversions on a C major seventh chord in closed position.¹⁶ Here, I define P-, R-, and L-equivalent

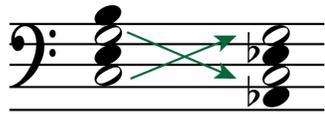
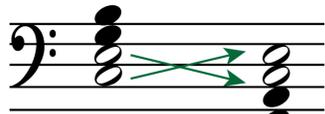
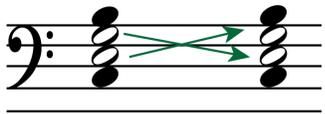
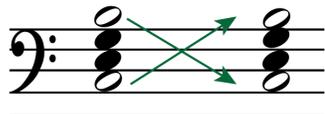
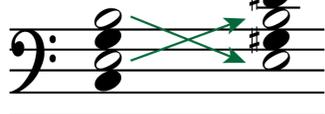
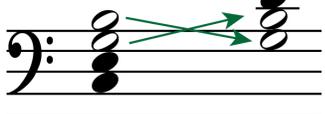
¹⁴ In the context of group theory, Gerald J. Balzano similarly describes the relationship between two triangles each representing major triads and minor triads. He writes of the triangles as a "set of adjoining triads alternately sharing m3 and M3 edges" (1980, 73).

¹⁵ For now, I am using contextual inversion as a representative for all conventional methods to highlight Strunk's geometric use of the Tonnetz. This is not to claim that contextual inversion defines the conventions of neo-Riemannian theory. However, a geometric interpretation of the Tonnetz is definitely not part of conventional neo-Riemannian theory.

¹⁶ It is also worth noting the similarity between these six contextual inversions with Gollin's six "edge-flips" (1998). If the same process of contextual inversions on the four notes of a seventh chord is applied to dominant seventh and half-diminished seventh chords, the result will be identical to Gollin's result. A dominant seventh chord type becomes a half-diminished chord type because the ordering of intervals is reversed upon inversion. The P-equivalent operation turns a C dominant seventh chord into an A half-diminished seventh chord; R-equivalent turns it into F# half-diminished; L-equivalent turns it into C# half-diminished; Root/7th invariant turns it into C half-diminished; 3rd/7th invariant turns it into E half-diminished; 5th/7th invariant turns it into G

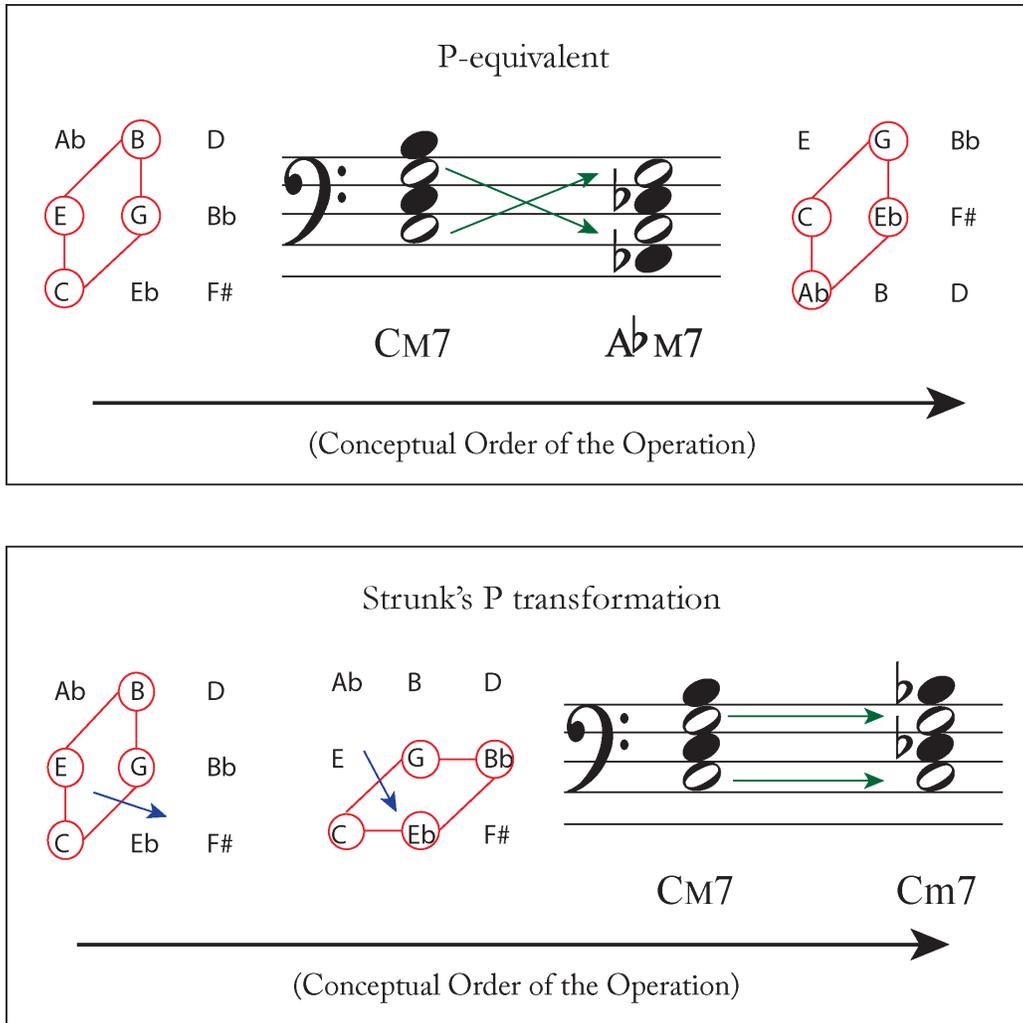
transformations according to the invariant intervals. P-equivalent transformations keep the perfect fifth between the root and the fifth, R-equivalent transformations keep the major third between the root and the third, and L-equivalent transformations keep the minor third between the third and the fifth. Each of the three P-, R-, and L-equivalent transformations contains the corresponding P, R, and L transformations on the C major triad. Since a seventh chord has four chord tones, three more contextual inversions are possible. One keeps the root and the seventh, another keeps the third and the seventh, and another keeps the fifth and the seventh. In contrast to the contextual inversions on major and minor triads, contextual inversions on major seventh and minor seventh chords do not change the type from one to the other because of the inversive symmetry of both types.

Figure 9. Contextual inversion of C major seventh chord.

P-equivalent	R-equivalent	L-equivalent
		
CM7 A \flat M7	CM7 FM7	CM7 CM7
Root/7th Invariant	3rd/7th Invariant	5th/7th Invariant
		
CM7 CM7	CM7 EM7	CM7 GM7

half-diminished. Gollin's "edge-flips" of a C dominant chord generate identical results (1998, 201).

Figure 10. Comparison of contextual inversion versus Strunk's reflection of a G-major seventh chord.



Now, let us compare the conventional method with Strunk's by applying the P-equivalent transformation and Strunk's P transformation on a C major seventh chord (Figure 10). Both transformations take the root and the fifth as invariant. The P-equivalent transformation generates an Ab major seventh chord as C and G exchange places through contextual inversion. At the pitch-class level, an ordered pitch-class set [0, 4, 7, e] becomes an ordered set [7, 3, 0, 8] and pitch classes 0 and 7 map onto each other reciprocally.¹⁷ The order of intervals remains

¹⁷ These pitch class sets are related by T_8 and T_7I because of intervallic symmetry. The T_7I relationship of the P-equivalent transformation is congruent with the T_7I relationship of the

unchanged because of the aforementioned intervallic symmetry of major seventh chord types. Although the chord is conceptually flipped upside down, the result of the contextual inversion is equivalent to transposition by a major third downward. Strunk's P transformation generates a C minor seventh chord because it holds interval class 5 invariant between C and G and reflects the geometric shape (parallelogram) over the C-G line segment. At the pitch-class level, an ordered pitch-class set [0, 4, 7, e] becomes another ordered set [0, 3, 7, t] where the dyad [0, 7] remains fixed. At the interval class level, reflection over a diagonal axis turns interval class 4 (connecting pitch classes C and E as well as G and B) into interval class 3 (connecting C and E \flat as well as G and B \flat). As a result, the chord quality changes from C major seventh to C minor seventh, and the set class changes from 4-20 (0158) to 4-26 (0358).

At the heart of this set-class alteration lies the mapping of interval class 4 (vertical line segment) into interval class 3 (horizontal line segment) as a result of the geometric reflection over a diagonal axis. This mapping plays a crucial role in all three of Strunk's transformations on the seventh chords because he considers the three transformations to be geometrically equivalent to a reflection over the diagonal axis followed by a translation, written as "Rf(D)Tr" (47, 50).¹⁸ This mapping is only possible because Strunk assigns the same length-as-Tonnetz distance (hereafter, "lengths") for line segments representing interval class 3 and interval class 4. As seen in Figure 4, the Tonnetz representation of each of the three transformations features two vertical line segments of a parallelogram ("a") mapping into two horizontal line segments of another parallelogram ("b"). The most important outcome of geometric treatment of the Tonnetz is that, even though the size of intervals is different in musical terms, one interval can map onto another by virtue of occupying the same distance on the Tonnetz. In other words, major thirds and minor thirds are two different intervals in music, but they are represented as equivalent on the Tonnetz.

conventional P transformation on a C major triad as noted by Strunk (Figure 3). R-equivalent and L-equivalent transformations on a C major seventh chord also feature the congruent T_nI relationships with R and L transformations on a C major triad (T_4I and $T_{11}I$ respectively).

¹⁸ For major and minor triads, Strunk uses two reflections (one over a diagonal axis and another over an anti-diagonal axis, notated as $2Rf(D)$) to maintain the same mapping of pitch classes as the equivalent inversions (50). The inversions are not contextual inversions since they are notated as T_7I , T_4I , and $T_{11}I$, corresponding to P, R, and L transformations (49).

classes 4-3 (0134) as seen in Figure 11.²⁰ The fact that both diagonal and anti-diagonal line segments occupy the same distance (because they are the two diagonals of a square) makes this mapping possible. However, the same mapping between interval class 1 and interval class 5 would not be possible under some different arrangement of pitch classes because the line segment that represents the interval class 1 may not be equal to that of the interval class 5. For example, in Figure 5, the length between E and E \flat is significantly longer than the length between C and G, whereas in Figure 3, the two intervals are represented as equal length. Geometric treatment of the Tonnetz, therefore, turns the lines connecting pitch classes from symbolic lines representing musical intervals into geometric lines.

The fact that the length of line segments determines the possibility for mapping intervals may inspire the construction of a new arrangement for a Tonnetz that features each line segment proportionally segmented according to the size of each interval. Such arrangement is not possible as observed by Candace Brower (2008). Brower observes that “an important difference between the geometry of visual ‘objects’ and that of musical ‘objects’...is [that] visual ‘objects’ have both height and width, but musical ‘objects’ have only height...To form a triangle with sides three, four and seven units in length, we would have to set the internal angles at 180° and 0°, which would cause the shorter sides to collapse onto the longer one” (65). Brower’s observation shows that non-proportional representation of the intervals is inevitable when forming a two-dimensional Tonnetz. Facing this inevitable discrepancy, Dmitri Tymoczko asks a series of framing questions about how to manage the discrepancy between the conventional treatment of the Tonnetz and a geometric one:

Considered as a *graph* [for a group-theoretic relationship], the Tonnetz is simply a collection of vertices and edges having no particular geometry or topology. To embed this graph into a robustly geometrical space requires us to ask questions like “should the ‘major third axis’ be a single straight line?” or “should the edges representing augmented triads be similar to those representing major and minor triads?” Our answers, rather than being simple consequences of the Tonnetz’s graph-theoretical structure, will depend on what we want to do with the space (2012, 38).

In light of the above quotations, Strunk’s method can be understood as an approach that takes advantage of the inherent discrepancy in the geometric use of the Tonnetz by recontextualizing the three transformations as geometric reflec-

²⁰ In each case, the net change of interval classes is the transformation of interval class 5 (diagonal line segments) into interval class 1 (anti-diagonal line segments).

tions so that the same transformation can represent the major-minor relationship regardless of the size of the chords.

Strunk's geometric recontextualization more closely reflects how jazz performers treat a chord. In jazz performance, the rhythm section plays major triads, major seventh chords, and major ninth chords interchangeably when they are built on the same root. Similarly, minor triads, minor seventh chords, and minor ninth chords are used just as interchangeably. Strunk writes, "Because of this interchangeability of voicings [in jazz performance practice], a correspondence between neo-Riemannian operations relating those triads, seventh chords, and ninth chords which function equivalently in jazz contexts is highly desirable" (Strunk 2003, 48).²¹ He comments on "recent publications" that apply neo-Riemannian theory to seventh chords, observing that they "do not discuss the functional equivalences desirable for the analysis of jazz" (Strunk 2003, 48n19).²² In other words, Strunk prefers a method that reflects jazz performance practice, in which the chord represented by a lead sheet symbol can be expressed with different voicings and extensions. The fact that a chord is a triad, a seventh chord, or a ninth chord is not too important as long as the chord is able to fulfill its functional role. Whereas the conventional (group-theoretic) method needs to develop a distinct set of transformations for triads, seventh chords, and ninth chords separately, Strunk's method does not treat each chordal extension as a different class of chords because, in jazz, the concept of "chord" is more malleable than in classical music. Understood in this way, Strunk's method of treating P, R, and L transformations as geometric reflection resonates with how a chord is treated in jazz performance practice.

REPRESENTING Z-RELATED SETS ON THE TONNETZ

On the Tonnetz, Strunk uses a parallelogram and its reflections over the two diagonal axes to represent major seventh and minor seventh chords, set class 4-20 (0158) and 4-26 (0358) respectively. As we observed earlier, the reflections of the parallelogram over the vertical and horizontal axes of the two chords will turn them into 4-7 (0145) and 4-3 (0134) respectively. This feature that one geometric shape can represent multiple sets can be extended to establish new

²¹ It is important to note that Strunk is not referring to harmonic function (i.e., Tonic, Subdominant, Dominant) but the fact that harmonies with the same core and different extensions (e.g., Am, Am7, Am9, Am11 etc.) are functionally interchangeable (i.e., "freely" interchangeable to fulfill the same function).

²² He cites Childs 1998, Douthett and Steinbach 1998, Bass 2001, and Fobes 2001 as the recent publications that "do not discuss the functional equivalences." All of them use contextual inversion and/or parsimonious voice-leading as a basis of their theories.

connections between sets as well as new ways to visualize previously established relationships between sets. This section explores the implications of representing *Z*-related pairs as two geometric shapes in reflections of each other. I have chosen *Z*-related sets because two *Z*-related sets share the same interval vector but belong to the different set classes. Since the arrangement of the Tonnetz that Strunk uses privileges the representation of interval classes 1, 3, 4, and 5 (as the four axes), not all of the *Z*-related pairs can be shown this way. However, a good number of them can be shown.

There are nineteen *Z*-related pairs of tetrachords, pentachords, and hexachords.²³ Among them, six *Z*-related pairs can be represented by two geometric shapes that are in geometric reflection with each other.²⁴ Consider, for example, 4-z15 (0146) and 4-z29 (0137), one of the *Z*-related pairs within this category. We can represent these two tetrachords as identical shapes in reflection over the horizontal axis because, upon reflection, interval class 1 (the antidiagonal line segment) and interval class 5 (the diagonal line segment) switch places, resulting in no net effect on the interval content of the chords (Figure 12). Figure 13 shows an excerpt from Alban Berg's "Warm die Lüfte," which uses the two *Z*-related tetrachords. The harmonic progression of the piano is represented on a Tonnetz in Figure 14. In addition, Figure 14 shows the excerpt's chromatic descent of 3-5 (016), the common trichordal subset of the pair, that sounds in the right hand. Each of these chords is represented as a "V" shape moving along the anti-diagonal axis from top left to bottom right.

²³ I chose to draw the line at the hexachord level because of two reasons. First, septa- and octachords are the inverse of penta- and tetrachords (respectively), which means, one can easily infer the Tonnetz relations of these larger sets from the representations of penta- and tetrachords similar to a photographic negative. Second, it is difficult to have meaningful visualizations of the larger sets when the Tonnetz representations become too complicated. I think the difficulty outweighs the usefulness.

²⁴ The six pairs in this category are: 4-z15/4-z29, 5-z17/5-z37, 5-z18/5-z38, 6-z6/6-z38, 6-z11/6-z40, and 6-z19/6-z44.

Figure 12. Tonnetz representation of 4-z15 and 4-z29

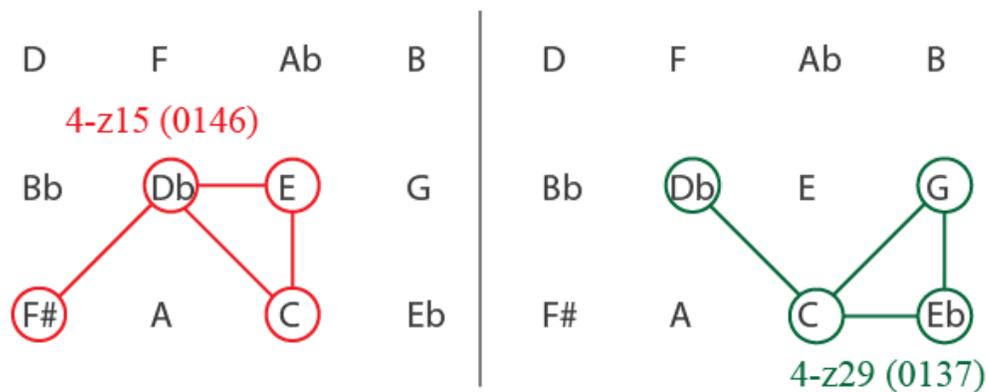


Figure 13. Alban Berg, 4 Gesänge, Op. 2, No. 4 "Warm die Lüfte," mm. 19–22.

The musical score shows the vocal line and piano accompaniment for the first four measures of the piece. The vocal line is marked *(tonlos)* and *p*. The piano accompaniment is marked *molto rit. ganz langsam* and *p dolce*. The lyrics are: "Stirb! Der Ei-ne stirbt, da-ne-ben der An-dre lebt." Below the piano part, six arrows point to specific chords, which are labeled as 4z-29, 4z-15, 4z-29, 4z-15, 4z-29, and 4z-15 from left to right.

with each other.²⁵ Consider, for example, 5-z12 (01356) and 5-z36 (01247). When mapped onto the Tonnetz, they take different shapes (Figure 15). For 5-z12, the shape can be interpreted as a square that consists of two diagonal and two anti-diagonal line segments (connecting pitch classes 0, 1, 5, and 6 in the figure) with one arm extending from the right vertex (connecting pitch classes 0 and 3 in the figure). Reflection over the horizontal axis will generate the same shape. Reflection over the vertical axis will also generate the same shape because the interval class 3 cycle (of which the extended arm is a part) loops around. Reflections over the diagonal and anti-diagonal axes will both generate a tetrachord 4-8 (0158) dissolving one component pitch class in each case. This is because the four nodes along the horizontal line that represent four distinct pitch classes in the interval class 3 cycle are turned into four nodes along the vertical line that represent three distinct pitch classes and one duplicate in the vertical interval class 4 cycle. For 5-z36, reflection over the vertical axis will generate the same shape. Reflection over the horizontal axis will generate the same shape flipped upside down, which is a mapping of a T_nI -related form of the original set. For example, in Figure 16, a reflection of an unordered pitch-class set $\{0,1,2,4,7\}$ over the horizontal line that connects the pitch classes t , 1, 4, and 7 will generate another unordered pitch-class set $\{1,4,6,7,8\}$ which is T_8I of the original set. Reflection over the diagonal axis will generate a pitch-class set from a different Z -related pair, 5-z37 (03458) and over the anti-diagonal axis will generate the other Z -related set of the same pair, 5-z17 (01348).

The remaining eight pairs (which are all hexachords) comprise four complexes of four hexachords each that are all related through a network of Z -relations and mirroring geometric shapes.²⁶ In Figure 17, 6-z3 (012356) and 6-z25 (013568) are represented as identical geometric shapes in reflection. Upon this reflection, the two interval-class-5 segments (i.e., 0-5 and 1-6) are transformed into interval-class-1 segments (i.e., 6-5 and 1-0) and vice versa. At the same time, their Z -related partners also share an identical geometric shape in reflection and involve an analogous interval-class swapping.²⁷

²⁵ The five pairs in this category are: 5-z12/5-z36, 6-z12/6-z41, 6-z17/6-z43, 6-z23/6-z45, and 6-z28/6-z49.

²⁶ The eight pairs in this category are: **6-z3**/6-z25:**6-z36**/6-z47, **6-z4**/6-z26:**6-z37**/6-z48, **6-z10**/6-z46:**6-z39**/6-z24, and **6-z13**/6-z50:**6-z29**/6-z42. Each pair on either side of a colon is represented by the same (but mirrored) geometric shape, and the set classes with the same typeface (i.e., both boldface or both regular) are Z related. For example, in the first complex, 6-z3 and 6-z25 are represented by mirroring shapes on the *Tonnetz*, while 6-z3 and 6-z36 are Z -related.

²⁷ Of course, one main reason why we can form complexes of two Z -related pairs is that the two hexachords of each Z -related pair are complements of each other.

Figure 16. Four Reflections of 5-z36 (01247).

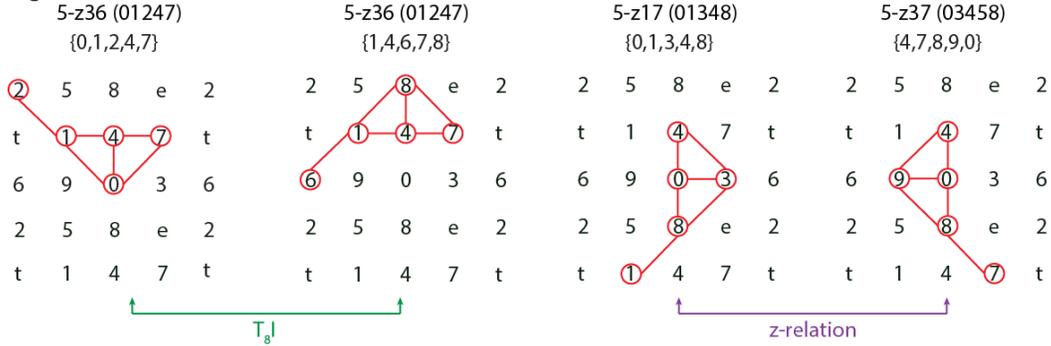
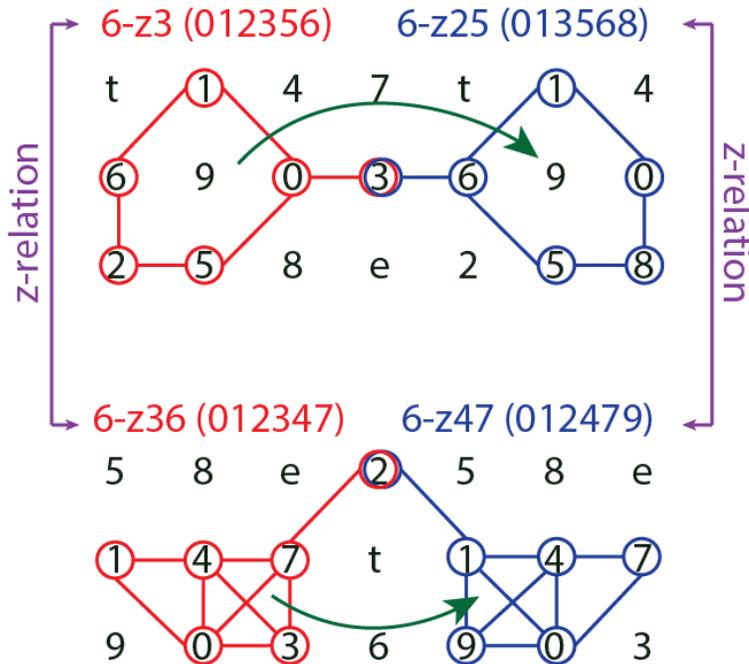


Figure 17. One of the four complexes of four hexachords related by mirroring shapes and Z relations.



Since the arrangement of pitch classes affects the geometric representation of pitch class sets on the Tonnetz, these geometric relationships between the Z-related pairs may not hold true in a Tonnetz that arranges pitch classes differently (such as the one presented in Figure 5). However, an analyst can arrange pitch classes differently or change the angles of vertices other than 90° to construct a new Tonnetz to target different pitch-class set relationships or specific composi-

tions.²⁸ A composer can also construct a new Tonnetz to use as what Robert Morris calls “compositional space,” which he describes as “out-of-time structures from which the more specific and temporally oriented compositional design can be composed” (1995, 330). In considering the numerous implications and potential applications that may arise from treating a Tonnetz in the way that Strunk did, the representation of Z-related sets on the Tonnetz is a small extension that warrants further research.

CONCLUSION

What is clear from a closer theoretical examination of Strunk's method is that the relationship between a musical sound and its spatial representation cannot be taken for granted. Whenever any musical sound is converted into some visualization, certain aspects of the sound are lost. On the other hand, visualizations can also bring out some new aspects of music that may not have been apparent only by listening. As more jazz theorists consider neo-Riemannian theory in analysis, Strunk's treatment of the Tonnetz reminds us that accounting for performance practice will enrich our understanding even further.

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²⁸ One such example is Joseph N. Straus's “Tonnetz for sc(014),” which he uses to analyze Anton Webern's *Concerto for Nine Instruments*, op. 24 and other post-tonal compositions (2011, 55).

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